# Idiosyncratic Information and Systematic Risk

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**Abstract**

When asset payoffs comprise systematic and idiosyncratic components, informed trading on idiosyncratic private information causes that information to be systematically reflected in the prices of other assets. Consequently, the risk associated with the realization of such information may be non-diversifiable. Cross-sectionally, information about assets with higher levels of systematic risk crowds out information about assets with lower levels of systematic risk. Trading on idiosyncratic information also generates excess volatility and excess comovement.

*Keywords*: asymmetric information; informed trading; systematic risk; idiosyncratic risk; asset pricing; excess volatility; excess covariance

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# 1 Introduction

Canonical asset pricing theory states that only systematic risk is relevant for pricing assets because idiosyncratic risk can be eliminated through diversification. Although this core principle applies in frictionless economies, it does not necessarily hold when traders are asymmetrically informed. As I demonstrate in this article, informed trading can transform idiosyncratic risk into systematic risk whereby idiosyncratic information is systematically reflected in the prices of other assets. Consequently, the risk related to the realization of idiosyncratic private information may, in certain situations, be non-diversifiable.

The channel through which idiosyncratic information is systematically incorporated into asset prices is a simple one: hedging. Trades made to exploit idiosyncratic information advantages effectively alter equilibrium allocations, which in turn affects traders' exposures to systematic risk. Such changes to equilibrium allocations and systematic risk exposures reflect traders' private information, so corresponding trades involving other assets intended to hedge changes to systematic risk exposures also reflect this information. Although the hedging demands of various traders may completely offset and aggregate to zero if all traders fully rebalance their portfolios following an information-based trade, the aggregate price impact on other assets may be nonzero because information asymmetry causes traders to have heterogeneous marginal rates of substitution. Thus, hedging transactions that originate from informed trading can propagate idiosyncratic information throughout the financial economy, causing it to be systematically reflected in prices.

Consider an illustrative example. Suppose there are two assets whose payoffs comprise a systematic component and an idiosyncratic component. Suppose further that there are two risk-averse traders: trader A is privately informed about asset 1's idiosyncratic component, whereas trader B is uninformed. If trader A observes a positive signal, then he increases his asset-1 allocation in proportion to the signal to take advantage of the information. This information-based trade raises his exposure to systematic risk, however, so he hedges that exposure by decreasing his demand for asset 2 proportional to the signal about asset 1. Because the market must clear in equilibrium, trader B's asset-1 allocation decreases by the same magnitude as trader A's asset-1 allocation increases, and his asset-2 demand increases by the same magnitude as trader A's asset-2 demand decreases. Hence, the traders' hedging demands completely offset, and trader A's private information about asset 1 has no effect on the price of asset 2 because the traders are symmetrically informed about asset 2 and, therefore, have identical marginal rates of substitution.

Now suppose there is a third risk-averse trader who is privately informed about asset 2's idiosyncratic component. If trader A observes a positive signal about asset 1, then trader B's and trader C's aggregate asset-1 allocation change and aggregate asset-2 hedging demand offset those of trader A. However, trader C's marginal rate of substitution for asset 2 differs from those of traders A and B because trader C is privately informed about asset 2. Consequently, the price impacts of the hedging trades do not fully offset one another, so trader A's private information about asset 1 is reflected in the price of asset 2.

This simple example illustrates the basic mechanism and highlights two critical elements that facilitate the systematic incorporation of idiosyncratic information into prices. First, idiosyncratic information about an asset must alter traders' demand for other assets in proportion to that information. Second, the traders' marginal rates of substitution must differ for the price impacts of their trades to not offset in aggregate. Both of these elements naturally arise from risk aversion and information asymmetry. Risk aversion incentivizes traders to hedge changes in their systematic risk exposures caused by information-based trades. Information asymmetry results in different levels of perceived risk among traders, which causes their marginal rates of substitution to differ. While risk aversion and information asymmetry are sufficient conditions for prices to systematically reflect idiosyncratic information, the phenomenon could also occur under risk neutrality. For instance, a risk-neutral institutional informed trader with a target portfolio beta would likely hedge its systematic risk exposure if an information-based trade altered the beta of its portfolio. Additionally, institutions often face portfolio constraints that could give rise to differing marginal rates of substitution, e.g., tax considerations, concentration limits, and liquidity needs.

I formalize the intuition described above in a tractable one-period setting with strategic informed trading. In the baseline model, there are two risk-averse informed traders and a third representative risk-averse uninformed trader. The financial economy comprises two risky stocks and a risk-free bond. Each stock payoff consists of a systematic component, which is perfectly correlated with the other stock's systematic component, and an idiosyncratic component, which is independent of the other stock's payoff. The two informed traders each possess private information about one stock's idiosyncratic component.

Each informed trader acts as an information monopolist and strategically controls both the price of the stock about which he possesses private information and the amount of his information conveyed through prices in equilibrium. Different from existing models of risk-averse strategic informed trading where asset payoffs do not include a systematic component, with systematic risk an informed trader must consider how idiosyncratic information about the other stock endogenously influences the price of the stock about which he is privately informed. Each informed trader also receives a nontradable endowment that camouflages his trading motive and, thus, prevents his information from being fully revealed by his trade.

In equilibrium, traders indirectly observe noisy signals of the informed traders' private information via the two stock prices. Equilibrium allocations and systematic risk exposures reflect these signals, so a change in allocations in one stock creates a proportional change in demand for the second stock, which causes the price of the second stock to reflect idiosyncratic information about the first stock, and vice versa. Thus, idiosyncratic information is systematically reflected in prices.

Idiosyncratic information about one stock has a greater influence on the other stock's price when there is more systematic risk. As the level of systematic risk increases, a marginal change in allocations has a bigger effect on risk exposures. Consequently, traders hedge their systematic risk exposures more aggressively, which amplifies the differences between the traders' marginal rates of substitution and increases the sensitivity of one stock's price to information about the other stock. In the limiting case with no systematic risk, idiosyncratic information is not reflected in the prices of other assets.

In an extension of the model that includes an additional stock and an additional informed trader, I analyze how multiple pieces of idiosyncratic information systematically influence prices of other assets. Information about a stock with a higher level of systematic risk has a greater influence on other prices than information about a stock with a lower level of systematic risk because, as discussed above, the aggressiveness with which traders hedge their risk exposures is increasing in a stock's level of systematic risk. Moreover, a higher level of systematic risk in one stock weakens another stock's informational influence on prices. This is because a stock with greater exposure to systematic risk provides a better hedge at the margin, which reduces the hedging demand for other stocks and, thereby, decreases other stocks' price impact.

An implication of these cross-sectional effects is that at least certain sources of idiosyncratic information should still be systematically reflected in prices even if there are many stocks on which informed trading occurs. Although the information signals in the model are independent and identically distributed, a large number of signals from many stocks would not offset one another (unless they all had the same level of systematic risk) because stocks with higher levels of systematic risk have their information reflected in prices to a greater degree than stocks with lower levels of systematic risk. With widespread informed trading, prices should mostly reflect information about stocks that have the highest levels of systematic risk.

The systematic incorporation of idiosyncratic information into prices is a robust phenomenon that occurs under a variety of circumstances. Notably, it is not necessary that an informed trader trade more than one asset for his information to be reflected in other prices, provided that other traders hedge their systematic risk exposures by trading that asset. When all traders trade every asset, however, whether idiosyncratic information about one asset is reflected in the price of another asset depends on whether traders are asymmetrically informed about the other asset. Without information asymmetry, traders face identical marginal rates of substitution for the other asset, so the price impacts of their hedging demands offset each other and aggregate to zero. Conversely, if traders information sets are asymmetric then there is a nonzero aggregate price impact because their marginal rates of substitution differ. Consequently, the other asset's price reflects idiosyncratic information in the latter case but not the former.

This article is related to several strands of literature, but perhaps most generally related to the literature that explores informed trading of multiple risky with correlated payoffs. Many models in this literature assume that privately informed traders trade in either a centralized market (Admati, 1985; Chan, 1993; Caballé and Krishnan, 1994; Bernhardt and Taub, 2008; Boulatov, Hendershott, and Livdan, 2013) or a decentralized market (He, 2009; Asriyan, Fuchs, and Green, 2017; Babus and Kondor, 2018) after observing a signal about an asset's total payoff, which includes both systematic and idiosyncratic components when asset payoffs are imperfectly correlated. Thus, by construction, a signal about one asset also conveys information about other assets, so equilibrium asset prices reflect all traders' private signals. My model differs from others in the literature, as here traders observe a signal only about an asset's idiosyncratic component rather than a signal that also includes a systematic component. Even though the traders' signals are independent of other asset payoffs, equilibrium prices nevertheless systematically reflect the traders' idiosyncratic information.

This article is also closely related to the financial contagion literature. While it is well established that hedging demand can cause idiosyncratic information about an asset (or liquidity) to be reflected in the price of another asset if one of the assets is traded by only a subset of traders (Fleming, Kirby, and Ostdiek, 1998; Kyle and Xiong, 2001; Cespa and Foucault, 2014; Gromb and Vayanos, 2018), traders in my model face no portfolio constraints and all assets are traded by all traders. Additionally, idiosyncratic information about one asset may be reflected in the price of another asset if traders also possess systematic information (King and Wadhwani, 1990; Pasquariello, 2006) or if the idiosyncratic information affects traders beliefs about other asset payoffs (Kodres and Pritsker, 2002). Correlated liquidity shocks may also create information spillovers (Calvo, 2004). However, in my model informed traders possess only idiosyncratic information about a single asset and their beliefs about other asset payoffs are unaffected by that information. Furthermore, their endowment shocks are uncorrelated. In other words, the only modeling ingredients are risk aversion and information asymmetry.

An additional implication of the model is that trading on idiosyncratic private information generates excess volatility and excess comovement because the idiosyncratic information that is systematically incorporated into prices is unrelated to other assets' fundamental values. There are other information-based explanations for excess volatility and comovement in the literature, as excess volatility can arise in the presence of ambiguity (Illeditsch, 2011) and excess comovement can occur when traders possess information that is correlated with other assets' fundamental values (Veldkamp, 2006; Mondria, 2010).

Informed trading is not the only context where idiosyncratic risk may influence asset prices. If markets are incomplete, asset prices may be affected by idiosyncratic labor shocks (Constantinides and Duffie, 1996; Heaton and Lucas, 1996; Storesletten, Telmer, and Yaron, 2007) and by idiosyncratic firm-level shocks that affect investment (Miao and Wang, 2007) and the value of human capital (Herskovic et al., 2016). Different from these models where idiosyncratic risk is important because traders cannot fully hedge that risk by trading other assets, idiosyncratic risk is important in my model because trading on idiosyncratic information causes the information to be systematically reflected in other prices and, thereby, transforms idiosyncratic risk associated with the information into systematic risk.

The remainder of the article is organized as follows. I describe the model in Section 2 and derive the equilibrium in Section 3. Cross-sectional pricing effects are analyzed in Section 4. Section 5 concludes. All proofs and a couple of simplified variations of the baseline model are in the appendix.

## 2 Model

The model is designed to illustrate in a tractable setting how informed trading causes prices to systematically reflect idiosyncratic information about other assets. The economy comprises two risky stocks in unit supply and an elastic supply of risk-free bonds with an interest rate of zero. Stock $i$ for $i \in \{1, 2\}$ generates a random payoff $\tilde{\pi}_i$ that consists of three components:

$$\tilde{\pi}_i = \beta_i \tilde{x} + \sqrt{1 - \beta_i^2} \tilde{y}_i + \tilde{z}_i, \tag{1}$$

where $\tilde{x} \sim \mathcal{N}(0, \sigma^2)$, $\tilde{y}_i \sim \mathcal{N}(0, \sigma^2)$, and $\tilde{z}_i \sim \mathcal{N}(0, \eta^2)$. The first component, $\tilde{x}$, affects both stock payoffs and represents the economy's systematic risk. Stock $i$'s exposure to this risk is denoted by $\beta_i \in (0, 1]$, with a bigger $\beta_i$ reflecting a higher level of systematic risk. The second component, $\tilde{y}_i$, affects only $\pi_i$ and ensures that stock $i$'s level of systematic risk does not affect its total risk. The third component, $\tilde{z}_i$, also affects only $\pi_i$ and is a source of idiosyncratic risk about which traders may be privately informed.

There are three traders, all of whom have identical preferences characterized by constant absolute risk aversion with risk tolerance $\frac{1}{\gamma}$. Two traders are privately informed, and one trader is uninformed. The first informed trader receives information about $\tilde{z}_1$, the second informed trader receives information about $\tilde{z}_2$, and the uninformed trader receives no private information about either stock. Specifically, trader $i$ observes the following signal:

$$f_i = \tilde{z}_i + \tilde{e}_i, \tag{2}$$

where $\tilde{e}_i \sim \mathcal{N}(0, \varepsilon^2)$. The information structure differs from most other models in the literature that analyze informed trading with correlated asset payoffs. In those models, information about one asset affects the prices of other assets by construction because the information is exogenously correlated with other asset payoffs. By contrast, informed traders here possess only idiosyncratic information that is uncorrelated with other asset payoffs. Consequently, the incorporation of idiosyncratic information about one asset into the price of another asset occurs endogenously.

Each trader is exogenously endowed with stock and bonds. Let $w_{ii}^s$, $w_{i\neg i}^s$, and $w_i^b$, respectively, denote informed trader $i$'s endowment of stock $i$, stock $\neg i$, and bonds. The uninformed trader's endowment of stock 1, stock 2, and bonds are denoted by $\hat{w}_1^s$, $\hat{w}_2^s$, and $\hat{w}^b$, respectively. Trader $i$'s equilibrium allocations to each stock and the bond are denoted by $s_{ii}$, $s_{i\neg i}$, and $b_i$, and the uninformed trader's allocations are denoted by $\hat{s}_1$, $\hat{s}_2$, and $\hat{b}$.

Additionally, each informed trader $i$ receives a nontradable random endowment denoted by $\tilde{q}_i \sim \mathcal{N}(0, \nu^2)$ which generates a payoff that, for simplicity, is perfectly correlated with $\tilde{z}_i$. To ensure well-defined solutions, the variance of the nontradable endowments must satisfy $\frac{\eta^2+\varepsilon^2}{\gamma^2\varepsilon^4} < \nu^2 < \frac{1}{\gamma^2\eta^2}$. This restriction states that trader $i$'s nontradable endowment must be sufficiently volatile to camouflage his private information but not so volatile that hedging his endowment shock is his primary motive for trade. The nontradable endowments are not publicly observable, but all other endowments are common knowledge. Furthermore, all random variables are mutually independent.

The sequence of events is portrayed in Figure 1. At date 1, all three traders receive their stock and bond endowments. The first informed trader privately observes $f_1$ and $q_1$, while the second informed trader privately observes $f_2$ and $q_2$. Then, trading in all assets occurs simultaneously. At date 2, stock payoffs are realized, portfolios are liquidated, and the traders consume their wealth.

# 3 Equilibrium

I conjecture and verify a unique affine equilibrium where each informed trader's private information about one of the stock payoffs in reflected in both stock prices. The informed traders are information monopolists and strategically control the amount of information conveyed by their trades through the prices. The price of stock $i$ is conjectured (and verified) to be an affine function of trader $i$'s demand and a noisy signal of trader $\neg i$'s private information:

$$p_i = \psi_i + \gamma\delta s_{ii} + \theta k_{\neg i}, \tag{3}$$

where $\psi_i$, $\delta$, and $\theta$ are constants and

$$k_i \equiv f_i - \gamma\varepsilon^2 q_i \tag{4}$$

is a noisy signal of trader $i$'s private information camouflaged by his nontradable endowment. The other two traders (trader $\neg i$ and the uninformed trader) can infer $k_i$ from the equilibrium stock prices. Given the symmetrical nature of the economy ($\pi_1$ and $\pi_2$, $f_1$ and $f_2$, and $q_1$ and $q_2$ are drawn from identical respective distributions), the constants $\delta$ and $\theta$ are the same for both stocks. The constants $\psi_1$ and $\psi_2$ generally differ, however, because they depend on $w_{11}^s$ and $w_{22}^s$.

The inclusion of the parameter $\theta$ in the price function is a novel feature and distinguishes the model from others in the literature that analyze strategic informed trading of a single asset. As discussed below, $\theta$ approaches zero in the limit as $\beta_i$ approaches zero. Hence, idiosyncratic information about stock $\neg i$ does not affect $p_i$ in the absence of systematic risk.

To obtain the equilibrium prices and allocations, I derive each trader's stock demand function and aggregate demand with supply to find the price. After privately observing the signal $f_i$, informed trader $i$ updates his beliefs about $\tilde{\pi}_i$ to

$$\tilde{\pi}_i | f_i \sim \mathcal{N}\left(\frac{\eta^2}{\eta^2 + \varepsilon^2} f_i\,,\,\Omega\right), \tag{5}$$

where

$$\Omega \equiv \sigma^2 + \frac{\sigma^2\varepsilon^2}{\sigma^2 + \varepsilon^2}. \tag{6}$$

Observation of the equilibrium prices similarly causes him to update his beliefs about $\tilde{\pi}_{\neg i}$. The uninformed trader also updates his beliefs about the stock payoffs by observing prices, and the posterior distribution of $\pi_i$ conditional on $k_i$ is

$$\tilde{\pi}_i | k_i \sim \mathcal{N}(\mu_i, \Sigma), \tag{7}$$

where

$$\mu_i \equiv \frac{\eta^2}{\Upsilon} k_i \tag{8}$$

$$\Sigma \equiv \sigma^2 + \frac{\eta^2 \varepsilon^2 (1 + \gamma^2 \varepsilon^2 \nu^2)}{\Upsilon} \tag{9}$$

$$\Upsilon \equiv \gamma^2 \varepsilon^4 \nu^2 + \eta^2 + \varepsilon^2. \tag{10}$$

Although each price contains two signals ($k_1$ and $k_2$), the traders can disentangle the signals by observing both prices simultaneously.

Informed trader $i$'s objective is to maximize his expected utility subject to a budget constraint given his private information $f_i$ and his inference about the other informed trader's private information, $k_{\neg i}$:

$$\max_{s_{i1}, s_{i2}} \quad \mathbb{E}\big[ - \exp[-\gamma \tilde{c}_i] \,|\, f_i, k_{\neg i} \big] \tag{11}$$

$$\text{s.t.} \quad c_i = b_i + s_{i1} \tilde{\pi}_1 + s_{i2} \tilde{\pi}_2 + q_i \tilde{z}_i \tag{12}$$

$$b_i = w_i^b + (w_{i1}^s - s_{i1}) p_1 + (w_{i2}^s - s_{i2}) p_2. \tag{13}$$

The first-order conditions to this maximization problem give trader $i$'s demand functions:

$$s_{ii} = \frac{\frac{\eta^2}{\eta^2 + \varepsilon^2} k_i - \theta k_{\neg i} - \psi_i + \gamma \delta w_{ii}^s}{\gamma(\Omega + 2\delta)} - \frac{\Sigma}{\Omega + 2\delta} \rho s_{i \neg i} \tag{14}$$

$$s_{i \neg i} = \frac{\mu_{\neg i} - p_{\neg i}}{\gamma \Sigma} - \rho s_{ii}. \tag{15}$$

where

$$\rho \equiv \frac{\sigma^2 \beta_1 \beta_2}{\Sigma} \tag{16}$$

denotes the conditional (on $k_i$ and $k_{\neg i}$) correlation between the two asset payoffs.

Trader $i$ adjusts his demand for each stock to account for his systematic risk exposure, which stems from the correlation in asset payoffs. Crucially, the degree to which an informed trader's allocation to a particular stock affects his demand for the other stock depends on whether he is privately informed about the stock. If he is informed, $\frac{\partial s_{ii}}{\partial s_{i\neg i}} = -\frac{\Sigma}{\Omega + 2\delta}\rho$; if he is uninformed, $\frac{\partial s_{i\neg i}}{\partial s_{ii}} = -\rho$. The different marginal rates of substitution, which arise from information asymmetry, result in a nonzero aggregate price impact from the various traders' hedging-based trades even though there is no change in aggregate systematic risk exposure. Consequently, idiosyncratic information about stock $\neg i$ is reflected in the price of stock $i$. By contrast, if all traders were symmetrically informed about an asset then the price of that asset would not reflect idiosyncratic information about other assets because the traders' marginal rates of substitution would be identical and, hence, the price impacts of their hedging-based trades would completely offset. Given that at least some degree of information asymmetry is likely to exist for most assets, however, the systematic incorporation of idiosyncratic information into prices is liable to be at least somewhat broad-based.

The uninformed trader's objective is to maximize his expected utility subject to a budget constraint and his inferences about the informed traders' private information, $k_1$ and $k_2$:

$$\max_{\hat{s}_1, \hat{s}_2} \quad \mathbb{E}\big[ -\exp[-\gamma \tilde{\hat{c}}] \,|\, k_1, k_2 \big] \tag{17}$$

$$\text{s.t.} \quad \hat{c} = \hat{b} + \hat{s}_1 \tilde{\pi}_1 + \hat{s}_2 \tilde{\pi}_2 \tag{18}$$

$$\hat{b} = \hat{w}^b + (\hat{w}_1^s - \hat{s}_1) p_1 + (\hat{w}_2^s - \hat{s}_2) p_2. \tag{19}$$

The corresponding first-order condition implies that his demand for stock $i$ is given by

$$\hat{s}_i = \frac{\mu_i - p_i}{\gamma\Sigma} - \rho\hat{s}_{\neg i}, \tag{20}$$

Like informed traders, the uninformed trader's demand for stock $i$ is influenced by his systematic risk exposure. Unlike informed traders, however, the degree to which the uninformed trader's allocation to a particular stock affects his demand for the other stock is the same for both stocks. Thus, the price impact of the uninformed trader's hedging-based trades does not offset the price impact of the informed traders' hedging-based trades.

The equilibrium prices are derived in the following way. Substituting $s_{ii}$ into (3) after solving the system of equations characterized by (14) and (15) gives expressions for $p_1$ and $p_2$ in terms of $\psi_1$, $\psi_2$, $\delta$, and $\theta$. Imposing the market-clearing condition,

$$s_{ii} + s_{\neg ii} + \hat{s}_i = 1, \tag{21}$$

gives alternative expressions for $p_1$ and $p_2$ that are also in terms of $\psi_1$, $\psi_2$, $\delta$, and $\theta$. Equating coefficients on $k_1$ and $k_2$ in the two sets of prices allows the equilibrium to be written in terms of the underlying parameters.

**Proposition 1.** *The unique equilibrium price of stock $i$ is given by*

$$p_i = \lambda_i + \xi k_i + \phi k_{\neg i}, \tag{22}$$

*where $\lambda_i$, $\xi$, and $\phi$ are defined in Appendix A.*

Proposition 1 shows that informed trader $i$'s idiosyncratic information about stock $i$ is reflected in the price of both stocks $i$ and $\neg i$. Hence, informed trading on idiosyncratic private information causes that information to be systematically reflected in the prices of other assets when asset payoffs also contain a systematic component. This implies that risk associated with the realization of such information may be non-diversifiable.

It is straightforward to show that $|\xi| > |\phi|$. Thus, although both prices incorporate idiosyncratic information about both stocks, information about stock $i$'s payoff has a bigger influence over the price of stock $i$ than information about stock $\neg i$. However, the relative degree to which prices reflect each piece of information depends on the aggregate amount of systematic risk, $B \equiv \beta_1\beta_2$. Define $\Gamma \equiv |\phi/\xi|$ as a normalized measure of price sensitivity to payoff-irrelevant information. In the absence of systematic risk, prices do not incorporate any idiosyncratic information pertaining to the payoffs of other assets: $\lim_{B\to 0}\Gamma = 0$. As the level of systematic risk rises, $\Gamma$ increases and the price of stock $i$ becomes relatively more sensitive to payoff-irrelevant information $k_{\neg i}$ compared to payoff-relevant information $k_i$.

Given the intricacy of the expressions for the equilibrium constants ($\delta$, $\theta$, $\xi$, $\phi$), analytic comparative statics of the equilibrium are not readily interpretable. I therefore numerically illustrate how changes in the level of systematic risk affect the unique equilibrium stock prices using the parameter values listed in Table 1. In Appendix B, I analyze a simplified version of the model where informed trader $i$ trades only stock $i$. Analytic comparative statics derived in that setting are consistent with the numerical results presented here.

Figure 2(a) plots $\Gamma$ as a function of $B$ and indicates that the relative sensitivity of stock $i$'s price to payoff-irrelevant idiosyncratic information about stock $\neg i$ is increasing in the level of systematic risk. As discussed above, the systematic incorporation of idiosyncratic information into prices arises from the traders' different marginal rates of substitution. Figure 2(b) plots the difference between the marginal rate of substitution for a trader who is informed about a stock and the marginal rate of substitution for a trader who is uninformed about that stock, $|1-\frac{\Sigma}{\Omega+2\delta}|\rho$, and shows that the difference is increasing in the level of systematic risk. Because there is a bigger price impact when there is a greater divergence in the traders' marginal rates of substitution, one stock's idiosyncratic information is reflected to a greater extent in the price of the other stock when $B$ is bigger.

The systematic incorporation of idiosyncratic information into prices also generates excess volatility unrelated to asset fundamentals, as is immediately apparent from (22). Although the level of excess volatility arising from informed trading is increasing in the amount of systematic risk ($\Gamma$ is increasing in $B$), the amount of systematic risk does not affect the traders' information sets because the noisy signal of trader $i$'s private information about stock $i$'s payoff, $k_i$, is unaffected by the amount of systematic risk and traders can infer each signal by observing both prices simultaneously. However, each price by itself becomes less informative as the level of systematic risk increases because the relative degree to which prices reflect payoff-irrelevant information is greater when there is a higher level of systematic risk.

In addition to influencing prices, systematic risk also affects the sensitivity of trader $i$'s equilibrium allocation to information about stock $\neg i$. While it is straightforward to show that $|\frac{\partial s_{ii}}{\partial k_i}| > |\frac{\partial s_{ii}}{\partial k_{\neg i}}|$, similar to prices the relative degree to which an informed trader's allocation reflects each piece of information depends on the amount of systematic risk. Define $\Theta \equiv |\frac{\partial s_{ii}}{\partial k_{\neg i}}|/|\frac{\partial s_{ii}}{\partial k_i}|$ as a normalized measure of demand sensitivity to payoff-irrelevant information. In the absence of systematic risk, allocations do not reflect any idiosyncratic information pertaining to the payoffs of other assets: $\lim_{B\to 0}\Theta = 0$. As the level of systematic risk increases, $\Theta$ increases and informed traders' allocations become more sensitive to payoff-irrelevant information compared to payoff-relevant information. This is simply because a marginal change in an informed trader's allocation to stock $\neg i$ has a bigger effect on his demand for stock $i$ when there is more systematic risk.

While a higher level of systematic risk increases the sensitivity of an informed trader's allocation to payoff-irrelevant information, it also increases the aggressiveness with which he exploits his own information advantage. Traders' demands for stocks about which they are uninformed become more sensitive to prices as systematic risk increases because they can adjust their exposure to the systematic risk factor $\tilde{x}$ by trading both assets (see (A.2) and (A.3) in Appendix A). Consequently, for the market to clear, the price reaction to an informed trade of a given size must be smaller when there is more systematic risk, and it can be shown that $\partial\delta/\partial B < 0$. This allows an informed trader to trade more aggressively on his information.

# 4 Cross-sectional pricing effects

The analysis in Section 3 demonstrates that prices systematically reflect idiosyncratic information in a setting with informed trading and two assets. Given the symmetry of the two stock payoffs in that setting, idiosyncratic information about each of the stocks has an identical influence over the other stock's price. In this section, I introduce a third asset with systematic risk exposure $\beta_3 \in (0, 1]$ to show how a stock's level of systematic risk affects its influence over other stock prices when the pairwise correlations among asset payoffs differ.

The third stock generates a payoff analogous to those of stocks 1 and 2:

$$\tilde{\pi}_3 = \beta_3 \tilde{x} + \sqrt{1 - \beta_3^2} \tilde{y}_3 + \tilde{z}_3, \tag{23}$$

where $\tilde{y}_3 \sim \mathcal{N}(0, \sigma^2)$ and $\tilde{z}_3 \sim \mathcal{N}(0, \eta^2)$. There is also a third informed trader who privately observes a noisy signal of $\tilde{z}_3$, analogous to (2). The traders' consumption functions and budget constraints are also modified to account for the third asset.

For expositional purposes, the three stocks and informed traders are referred to as $i$, $j$, and $\neg j$, with $i$ denoting the stock and informed trader of focus and $j$ and $\neg j$ denoting the other two stocks and informed traders. Similar to the setting with two stocks, the price of stock $i$ is conjectured to be an affine function of trader $i$'s demand and noisy signals of trader $j$'s and $\neg j$'s private information:

$$p_i = \psi_i + \gamma \delta_i s_{ii} + \theta_{ij} k_j + \theta_{i \neg j} k_{\neg j}. \tag{24}$$

With more than two stocks, the equilibrium is no longer symmetric. Accordingly, the sensitivity of each price to information about other stocks depends on the pairwise correlations between the stock payoffs, and these correlations depend on the stocks' $\beta$'s. Because the price sensitivity to information varies across stocks, the sensitivity of price $i$ to trader $i$'s demand also depends on the $\beta$'s. Note that the sensitivity of price $j$ to information about stock $\neg j$

generally differs from the sensitivity of price $\neg j$ to information about stock $j$ because the correlation between stocks $i$ and $j$ generally differs from the correlation between stocks $i$ and $\neg j$. Hence, $\theta_{j\neg j} \neq \theta\neg jj$. This means that there are a total of 24 equilibrium constants (3 $\psi$'s, $\delta$'s, $\lambda$'s, and $\xi$'s and 6 $\theta$'s and $\phi$'s).

Informed trader $i$'s demand functions are given by

$$s_{ii} = \frac{\frac{\eta^2}{\eta^2+\varepsilon^2}k_i - \theta_{ij}k_j - \theta_{i\neg j}k_{\neg j} - \psi_i + \gamma\delta_i w_{ii}^s}{\gamma(\Omega + 2\delta_i)} - \frac{\Sigma}{\Omega + 2\delta_i}(\rho_{ij}s_{ij} + \rho_{i\neg j}s_{i\neg j}) \tag{25}$$

$$s_{ij} = \frac{\mu_j - p_j}{\gamma\Sigma} - \rho_{ij}s_{ii} - \rho_{j\neg j}s_{i\neg j} \tag{26}$$

$$s_{i\neg j} = \frac{\mu_{\neg j} - p_{\neg j}}{\gamma\Sigma} - \rho_{i\neg j}s_{ii} - \rho_{j\neg j}s_{ij}, \tag{27}$$

where $\rho_{ij}$ denotes the conditional pairwise correlation between stocks $i$ and $j$, analogous to (16). Like the economy with only two stocks, trader $i$'s marginal rate of substitution for a stock about which he is informed differs from his marginal rate of substitution for a stock about which he is uninformed. This causes the price of stock $i$ to systematically reflect idiosyncratic information about the other two stocks, but the relative degree to which idiosyncratic information about stocks $j$ and $\neg j$ influences the price of stock $i$ depends on their $\beta$'s. Recall from the discussion in Section 3 that information asymmetry creates a bigger divergence in the traders' marginal rates of substitution when the correlation between the payoffs is higher. Because the price impact is increasing in the difference between the traders' marginal rates of substitution, information about stocks with higher levels of systematic risk is reflected to a greater extent in other prices than information about stocks with lower levels of systematic risk.

The equilibrium prices are derived in a similar fashion as those in Proposition 1. Imposing a market-clearing condition analogous to (21) and substituting trader $i$'s demand for stock $i$ into (24) generates a system of 24 non-linear equations in the 24 unknown equilibrium constants. Given the complexity of the system, closed form analytic solutions are not attainable.

Nonetheless, the system of equations—and, hence, the equilibrium—can be solved numerically. The numerical results presented here are consistent with analytic results obtained from a simplified version of the three-stock economy analyzed in Appendix C.

**Proposition 2.** *There exists an equilibrium where the price of stock $i$ is given by*

$$p_i = \lambda_i + \xi_i k_i + \phi_{ij} k_j + \phi_{i\neg j} k_{\neg j}. \tag{28}$$

Proposition 2 shows that prices systematically reflect idiosyncratic information about multiple assets when informed traders trade on the basis of that information. While the numerical solution confirms the existence of an equilibrium, it does not guarantee uniqueness. However, the equilibrium is unique when $\beta_3 = 0$ because information about the third stock does not influence the prices of the other two stocks, and vice versa, if the third stock has no systematic risk. By continuity, any equilibrium other than the one presented in Figure 3 (if multiple equilibria exist) should result in similar cross-sectional effects, at least for small and moderate levels of systematic risk.

Figure 3(a) depicts the relative sensitivity of $p_i$ to idiosyncratic information about stocks $j$ and $\neg j$ ($\Gamma_{ij} \equiv |\phi_{ij}/\xi_i|$ and $\Gamma_{i\neg j} \equiv |\phi_{i\neg j}/\xi_i|$) as a function of $\beta_i$ when $\beta_j = 1$ and $\beta_{\neg j} = 0.5$. Consistent with the two-asset economy analyzed in Section 3, the degree to which the price of stock $i$ reflects information about stocks $j$ and $\neg j$ is increasing in $\beta_i$. Hence, a stock with a higher level of systematic risk is more sensitive to idiosyncratic information about other assets.

The more interesting cross-sectional effects are in relation to $\beta_j$. Because a change in a trader's allocation to a high-$\beta$ stock has a bigger effect on a his systematic risk exposure than an equivalent change in his allocation to a low-$\beta$ stock, he hedges the former allocation change more aggressively than the latter. This causes information about a high-$\beta$ stock to have a bigger effect on $p_i$ than information about a low-$\beta$ stock. Thus, the extent to which $p_i$ reflects information about stock $j$ is increasing in $\beta_j$, as shown in Figure 3(b), which depicts

$\Gamma_{ij}$ as a function of $\beta_j$ when $\beta_i = 1$ and $\beta_{\neg j} = 0.5$.

Conversely, the extent to which $p_i$ reflects information about stock $\neg j$ is decreasing in $\beta_j$. As $\beta_j$ increases, a marginal change in a trader's allocation to stock $j$ has a bigger effect on his systematic risk exposure. This increases the relative effectiveness of using stock $j$ to hedge his exposure, which decreases his hedging demand for stock $\neg j$. Consequently, information about stock $\neg j$ has a smaller influence on $p_i$ when $\beta_j$ is larger, as shown in Figure 3(c). In essence, information about high-$\beta$ stocks crowds out information about low-$\beta$ stocks.

The crowding-out effect implies that information about stocks with higher levels of systematic risk will overshadow information about stocks with lower levels of systematic risk when there are many stocks on which informed trading occurs. Thus, although the individual $k_i$'s are independent and identically distributed, the pieces of idiosyncratic information generally will not offset one another even with many assets because the coefficients on the $k_i$'s differ. Proposition 2 also suggests that trading on idiosyncratic private information can generate excess comovement because information about one stock is systematically reflected in the prices of multiple other stocks, but the greatest excess comovement occurs for stocks with high levels of systematic risk.

Additionally, Figure 3(c) suggests that prices will systematically reflect idiosyncratic information about other assets as long as the number of assets is finite. Note that the limiting case where $\beta_j$ approaches zero corresponds to a setting where only stock $\neg j$ can be used to hedge changes in systematic risk exposure that arise from trading stock $i$.[1] Hence, stock $j$'s impact on $\Gamma_{i\neg j}$ represents the degree to which the existence of an additional asset (that traders can use to hedge their systematic risk exposure) affects the systematic incorporation of other idiosyncratic information into prices. Although an additional asset reduces the sensitivity of prices to idiosyncratic information, the magnitude of the effect is modest, especially for small and moderate values of $\beta_j$. Moreover, the marginal impact of an additional asset should be decreasing in the number of assets because each additional asset proves a smaller

[1] The magnitude of $\lim_{\beta_j \to 0} \Gamma_{i\neg j}$ is slightly smaller than that of $\lim_{\beta_i \to 0} \Gamma$ because there is now a third informed trader.

marginal hedging benefit.

# 5 Conclusion

Trading on idiosyncratic private information causes that information to be systematically reflected in the prices of other assets when risk-averse traders are asymmetrically informed about those assets. Risk aversion incentivizes traders to hedge changes to their systematic risk exposures arising from information-based trades, and information asymmetry prevents the price impacts of those trades from fully offsetting one another because asymmetrically-informed traders have different marginal rates of substitution. Although risk aversion and information asymmetry are sufficient conditions for prices to systematically reflect idiosyncratic information, the effect could occur among risk-neutral traders who have target portfolio betas and different marginal rates of substitution.

Another requirement for prices to systematically reflect idiosyncratic information is that the number of assets must be a finite. If traders could hedge changes to their systematic risk exposures with infinitely many assets, then the price impact on each asset would approach zero. Accordingly, the effect should be strongest for subsets of assets that are highly correlated with each other but that have a low correlation with the broader market. Such assets provide an effective hedge for risk that cannot be hedged with other assets, so traders will more aggressively hedge their risk exposures with those assets.

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# Appendix A: Proofs

## Proof of Proposition 1

Solving the system of equations characterized by (14) and (15) yields

$$s_{ii} = \frac{\frac{\eta^2}{\eta^2+\varepsilon^2}k_i - \psi_i + \gamma\delta w_{ii}^s - \rho\left(\frac{\eta^2}{\Upsilon}k_{\neg i} - p_{\neg i}\right) - \theta k_{\neg i}}{\gamma(\Omega + 2\delta - \Sigma\rho^2)} \tag{A.1}$$

$$s_{i\neg i} = \frac{\frac{\Omega+2\delta}{\Sigma}(\mu_{\neg i} - p_{\neg i}) - \rho\left(\frac{\eta^2}{\eta^2+\varepsilon^2}k_i - \psi_i + \gamma\delta w_{ii}^s - \theta k_{\neg i}\right)}{\gamma(\Omega + 2\delta - \Sigma\rho^2)}. \tag{A.2}$$

Substituting (A.1) into (3) gives

$$\begin{aligned} p_i = \psi_i &- \frac{\gamma\Sigma\rho(1+\rho)\delta}{2(\Omega + 2\delta - \Sigma\rho^2) + (\Omega + 2\delta - \Sigma)\rho} \\ &- \frac{[4(\Omega + 2\delta - \Sigma\rho^2) - (\Omega + 2\delta - \Sigma)\rho^2]\delta(\psi_i - \gamma\delta w_{ii}^s) + 2\Sigma\rho(1-\rho^2)\delta(\psi_{\neg i} - \gamma\delta w_{\neg i\neg i}^s)}{4(\Omega + 2\delta - \Sigma\rho^2)^2 - (\Omega + 2\delta - \Sigma)^2\rho^2} \\ &+ \frac{[(\Omega + 2\delta)(1-\rho^2) + 3(\Omega + 2\delta - \Sigma\rho^2)]\frac{\eta^2}{\eta^2+\varepsilon^2} - 2\Sigma\rho(1-\rho^2)\theta}{4(\Omega + 2\delta - \Sigma\rho^2)^2 - (\Omega + 2\delta - \Sigma)^2\rho^2}\delta k_i \\ &+ \left(\theta + \frac{2\Sigma\rho(1-\rho^2)\frac{\eta^2}{\eta^2+\varepsilon^2} - [4(\Omega + 2\delta - \Sigma\rho^2) - (\Omega + 2\delta - \Sigma)\rho^2]\theta}{4(\Omega + 2\delta - \Sigma\rho^2)^2 - (\Omega + 2\delta - \Sigma)^2\rho^2}\delta\right)k_{\neg i}. \end{aligned}$$

Solving the system of equations characterized by (20) yields

$$\hat{s}_i = \frac{\mu_i - p_i - (\mu_{\neg i} - p_{\neg i})\rho}{\gamma\Sigma(1-\rho^2)}. \tag{A.3}$$

Imposing the market-clearing condition and solving the resulting system of equations gives (22), where

$$\begin{aligned} \lambda_i &\equiv -\frac{\gamma\Sigma(1+\rho)(\Omega + 2\delta - \Sigma\rho^2)}{2(\Omega + 2\delta - \Sigma\rho^2) + (\Omega + 2\delta - \Sigma)\rho} \\ &\quad - \frac{\Sigma(1-\rho^2)[2(\Omega + 2\delta - \Sigma\rho^2)(\psi_i - \gamma\delta w_{ii}^s) - (\Omega + 2\delta - \Sigma)\rho(\psi_{\neg i} - \gamma\delta w_{\neg i\neg i}^s)]}{4(\Omega + 2\delta - \Sigma\rho^2)^2 - (\Omega + 2\delta - \Sigma)^2\rho^2} \\ \xi &\equiv \frac{\eta^2}{\Upsilon} + \frac{\Sigma(1-\rho^2)[2(\Omega + 2\delta - \Sigma\rho^2)\frac{\eta^2}{\eta^2+\varepsilon^2} + (\Omega + 2\delta - \Sigma)\rho\theta]}{4(\Omega + 2\delta - \Sigma\rho^2)^2 - (\Omega + 2\delta - \Sigma)^2\rho^2} \\ \phi &\equiv -\frac{\Sigma(1-\rho^2)[2(\Omega + 2\delta - \Sigma\rho^2)\theta + (\Omega + 2\delta - \Sigma)\rho\frac{\eta^2}{\eta^2+\varepsilon^2}]}{4(\Omega + 2\delta - \Sigma\rho^2)^2 - (\Omega + 2\delta - \Sigma)^2\rho^2}. \end{aligned}$$

Equating the coefficients on $k_i$ and $k_{\neg}i$ in the two price expressions yields

$$\psi_i \equiv -\frac{\gamma\Sigma(1+\rho)(\Omega+2\delta-\Sigma\rho^2-\delta\rho)}{2\Omega+2\delta+\Sigma+(\Omega+\delta-\Sigma)\rho-3\Sigma\rho^2} - \frac{\gamma\Sigma\rho(1-\rho^2)(\Omega+4\delta-\Sigma)\delta w^s_{\neg i\neg i}}{[2\Omega+2\delta+\Sigma(1-3\rho^2)]^2-(\Omega+\delta-\Sigma)^2\rho^2}$$
$$-\frac{\big((\Omega+2\delta-\Sigma\rho^2)^2-(\Omega+\delta-\Sigma)^2\rho^2-(\Sigma-\Omega)\rho^2\delta-[(\Omega+\Sigma)^2-4\Sigma^2\rho^2](1-\rho^2)\big)\gamma\delta w^s_{ii}}{[2\Omega+2\delta+\Sigma(1-3\rho^2)]^2-(\Omega+\delta-\Sigma)^2\rho^2}$$
$$\theta \equiv -\frac{\eta^2}{\eta^2+\varepsilon^2}\left(1-\frac{[(\Omega+\delta-\Sigma\rho)(1+\rho)+\Omega+\delta+\Sigma(1-2\rho^2)][\Omega+2\delta+\Sigma\rho)(1-\rho)+\Omega+2\delta-\Sigma\rho^2]}{2(\Omega+2\delta-\Sigma\rho^2)^2+2(\Omega+2\delta)(\Omega+\Sigma)-(\Omega+\delta+\Sigma)(\Omega+2\delta+3\Sigma)\rho^2+4\Sigma^2\rho^4}\right),$$

and $\delta$ is the unique positive solution to the quartic polynomial (economically, $\delta$ must be positive):

$$\begin{aligned}
0 = &[(\Omega+2\delta)(2+\rho)-\Sigma\rho(1+2\rho)][(\Omega+2\delta)(2-\rho)+\Sigma\rho(1-2\rho)]\\
&\times\big[(4-\rho^2)(\gamma^2\varepsilon^4\nu^2-\eta^2-\varepsilon^2)\delta^2\\
&\quad+\big((\gamma^2\varepsilon^4\nu^2-2\eta^2-2\varepsilon^2)[\Omega(1-\rho^2)+3(\Omega-\Sigma\rho^2)]-4(\eta^2+\varepsilon^2)\Sigma(1-\rho^2)\big)\delta\\
&\quad+\Upsilon\Sigma[3\Sigma\rho^2(1-\rho^2)-(2\Omega+\Sigma)(1-\rho^2)]\\
&\quad+(\eta^2+\varepsilon^2)\big((\Omega^2+\Sigma^2)\rho^2-4(\Omega-\Sigma\rho^2)^2-2\Sigma[\Omega-\Sigma\rho^2(1-\rho^2)]\big)\big].
\end{aligned}$$

□

## Proof of Proposition B.1

The proof is similar to the proof of Proposition 1. Substituting informed trader $i$'s demand into (24) gives

$$p_i = \frac{(\Omega+\bar{\delta})\bar{\psi}_i+\bar{\delta}^2 w^s_i}{\Omega+2\bar{\delta}} + \frac{\frac{\eta^2}{\eta^2+\varepsilon^2}\bar{\delta}}{\Omega+2\bar{\delta}}k_i + \frac{(\Omega+\bar{\delta})\bar{\theta}}{\Omega+2\bar{\delta}}k_{\neg i}, \tag{A.4}$$

and imposing the market-clearing condition gives

$$p_i = \mu_i + \frac{\Sigma(1-\rho^2)}{\Omega+2\bar{\delta}}\Big(\frac{\eta^2}{\eta^2+\varepsilon^2}k_i - \bar{\theta}k_{\neg i} - \gamma(\Omega+2\bar{\delta}) - \bar{\psi}_i + \gamma\bar{\delta}w^s_i\Big) - (\mu_{\neg i}-p_{\neg i})\rho. \tag{A.5}$$

Solving the system of equations characterized by (A.5) yields (B.2), where

$$\bar{\lambda}_i \equiv -\gamma\Sigma\Big(1+\rho+\frac{\bar{\psi}_i-\gamma\bar{\delta}w_i^s+(\bar{\psi}_{\neg i}-\gamma\bar{\delta}w_{\neg i}^s)\rho}{\gamma(\Omega+2\bar{\delta})}\Big)$$

$$\bar{\xi} \equiv \frac{\eta^2}{\Upsilon}+\frac{\Sigma\big(\frac{\eta^2}{\eta^2+\varepsilon^2}-\theta\rho\big)}{\Omega+2\bar{\delta}}$$

$$\bar{\phi} \equiv \frac{\Sigma\big(\frac{\eta^2}{\eta^2+\varepsilon^2}\rho-\theta\big)}{\Omega+2\delta}.$$

Then, equating the coefficients on $k_i$ and $k_{\neg i}$ in (A.4) and (B.2) gives

$$\bar{\psi}_i \equiv \frac{\gamma[(\Sigma-\bar{\delta})(\Omega+\bar{\delta}+\Sigma)-\Sigma^2\rho^2]\bar{\delta}w_i^s+\gamma\Sigma(\Omega+2\bar{\delta})\bar{\delta}\rho w_{\neg i}^s}{(\Omega+\bar{\delta}+\Sigma)^2-\Sigma^2\rho^2}-\frac{\gamma\Sigma(\Omega+2\bar{\delta})(1+\rho)}{\Omega+\bar{\delta}+\Sigma(1+\rho)}$$

$$\bar{\delta} \equiv \frac{\gamma^2\varepsilon^4\nu^2(\Omega+2\Sigma)+\sqrt{\gamma^4\varepsilon^8\nu^4(\Omega+2\Sigma)^2-4(\gamma^2\varepsilon^4\nu^2-\eta^2-\varepsilon^2)\Upsilon\Sigma^2\rho^2}}{2(\gamma^2\varepsilon^4\nu^2-\eta^2-\varepsilon^2)}-(\Omega+\Sigma)$$

$$\bar{\theta} \equiv \frac{\big(\gamma^2\varepsilon^4\nu^2(\Omega+2\Sigma)-\sqrt{\gamma^4\varepsilon^8\nu^4(\Omega+2\Sigma)^2-4(\gamma^2\varepsilon^4\nu^2-\eta^2-\varepsilon^2)\Upsilon\Sigma^2\rho^2}\big)\frac{\eta^2}{\eta^2+\varepsilon^2}}{2\Upsilon\Sigma\rho}.$$

□

## Proof of Corollary B.1

The partial derivatives of $\bar{\theta}$ and $\bar{\delta}$ with respect to $B$ are

$$\frac{\partial\bar{\theta}}{\partial B}=\frac{\gamma^2\sigma^2\varepsilon^4\nu^2(\Omega+2\Sigma)\bar{\theta}}{\Sigma\rho\sqrt{\gamma^4\varepsilon^8\nu^4(\Omega+2\Sigma)^2-4(\gamma^2\varepsilon^4\nu^2-\eta^2-\varepsilon^2)\Upsilon\Sigma^2\rho^2}}>0$$

$$\frac{\partial\bar{\delta}}{\partial B}=-\frac{2\sigma^2\Upsilon\Sigma\rho}{\sqrt{\gamma^4\varepsilon^8\nu^4(\Omega+2\Sigma)^2-4(\gamma^2\varepsilon^4\nu^2-\eta^2-\varepsilon^2)\Upsilon\Sigma^2\rho^2}}<0.$$

The partial derivative of $\bar{\Gamma}$ with respect to $B$ is

$$\frac{\partial\bar{\Gamma}}{\partial B}=\frac{(\Omega+\bar{\delta})\bar{\delta}\frac{\partial\bar{\theta}}{\partial B}-\Omega\bar{\theta}\frac{\partial\bar{\delta}}{\partial B}]}{\frac{\eta^2}{\eta^2+\varepsilon^2}\bar{\delta}^2}>0,$$

and the partial derivative of $\bar{\Theta}$ with respect to $B$ is

$$\frac{\partial \bar{\Theta}}{\partial B} = \Big(\frac{\eta^2+\varepsilon^2}{\eta^2}\Big)\frac{\partial \bar{\theta}}{\partial B} > 0.$$

□

# Proof of Proposition C.1

The uninformed trader's demand functions are given by

$$\begin{aligned}
\hat{s}_1 &= \frac{\mu_1 - p_1 - \gamma\sigma^2(\beta_2\hat{s}_2 + \beta_3\hat{s}_3)\beta_1}{\gamma\Sigma} \\
\hat{s}_2 &= \frac{\mu_2 - p_2 - \gamma\sigma^2(\beta_1\hat{s}_1 + \beta_3\hat{s}_3)\beta_2}{\gamma\Sigma} \\
\hat{s}_3 &= -\frac{p_3 + \gamma\sigma^2(\beta_1\hat{s}_1 + \beta_2\hat{s}_2)\beta_3}{\gamma(\sigma^2+\eta^2)}.
\end{aligned}$$

Imposing the market-clearing condition gives

$$\begin{aligned}
p_1 = \mu_1 &- \frac{[\eta^2+\sigma^2(1-\beta_3^2)]\sigma^2\beta_1\beta_2}{(\sigma^2+\eta^2)\Sigma - \sigma^4\beta_2^2\beta_3^2}(\mu_2 - p_2) + \frac{(\Sigma-\sigma^2\beta_2^2)\sigma^2\beta_1\beta_3 p_3}{(\sigma^2+\eta^2)\Sigma - \sigma^4\beta_2^2\beta_3^2} \\
&+ \frac{\Sigma - \frac{[\eta^2+\sigma^2(1-\beta_3^2)]\sigma^4\beta_1^2\beta_2^2 + (\Sigma-\sigma^2\beta_2^2)\sigma^4\beta_1^2\beta_3^2}{(\sigma^2+\eta^2)\Sigma-\sigma^4\beta_2^2\beta_3^2}}{\Omega+2\delta}\Big(\frac{\eta^2}{\eta^2+\varepsilon^2}k_1 - \theta k_2 - \gamma(\Omega+2\delta) - \psi_1' + \gamma\delta w_1^s\Big) \\
p_2 = \mu_2 &- \frac{[\eta^2+\sigma^2(1-\beta_3^2)]\sigma^2\beta_1\beta_2}{(\sigma^2+\eta^2)\Sigma - \sigma^4\beta_1^2\beta_3^2}(\mu_1 - p_1) + \frac{(\Sigma-\sigma^2\beta_1^2)\sigma^2\beta_2\beta_3 p_3}{(\sigma^2+\eta^2)\Sigma - \sigma^4\beta_1^2\beta_3^2} \\
&+ \frac{\Sigma - \frac{[\eta^2+\sigma^2(1-\beta_3^2)]\sigma^4\beta_1^2\beta_2^2 + (\Sigma-\sigma^2\beta_1^2)\sigma^4\beta_2^2\beta_3^2}{(\sigma^2+\eta^2)\Sigma-\sigma^4\beta_1^2\beta_3^2}}{\Omega+2\delta}\Big(\frac{\eta^2}{\eta^2+\varepsilon^2}k_2 - \theta k_1 - \gamma(\Omega+2\delta) - \psi_2' + \gamma\delta w_2^s\Big) \\
p_3 = -\gamma&\Big(\sigma^2+\eta^2 - \frac{[(\beta_1^2+\beta_2^2)\Sigma - 2\sigma^2\beta_1^2\beta_2^2]\sigma^4\beta_3^2}{\Sigma^2 - \sigma^4\beta_1^2\beta_2^2}\Big) \\
&- \frac{(\Sigma-\sigma^2\beta_2^2)\sigma^2\beta_1\beta_3}{\Sigma^2-\sigma^4\beta_1^2\beta_2^2}(\mu_1 - p_1) - \frac{(\Sigma-\sigma^2\beta_1^2)\sigma^2\beta_2\beta_3}{\Sigma^2-\sigma^4\beta_1^2\beta_2^2}(\mu_2 - p_2),
\end{aligned}$$

and solving this system of equations yields (C.1) along with

$$p_i = -\gamma\Big(\Sigma + \sigma^2(\beta_{\neg i} + \beta_3)\beta_i + \frac{[\psi_i' - \gamma\delta w_i^s + (\psi_{\neg i}' - \gamma\delta w_{\neg i}^s)\rho]\Sigma}{\Omega + 2\delta}\Big) + \mu_i + \frac{\Sigma(\frac{\eta^2}{\eta^2+\varepsilon^2} - \theta\rho)}{\Omega + 2\delta}k_i - \frac{\Sigma(\theta - \frac{\eta^2}{\eta^2+\varepsilon^2}\rho]}{\Omega + 2\delta}k_{\neg i}.$$

The equilibrium constants are obtained by the process described in the proof of Proposition 1. □

## Proof of Corollary C.1

Consider the sensitivity of stock 3's price to idiosyncratic information about stock $i$:

$$\frac{\partial p_3}{\partial k_i} = \frac{\sigma^2\big(\frac{\eta^2}{\eta^2+\varepsilon^2}\beta_i - \theta\beta_{\neg i}\big)\beta_3}{\Omega + 2\delta}. \tag{A.6}$$

Note that $\lim_{\beta_i \to 0} \frac{\partial p_3}{\partial k_i} = 0$. Differentiating (A.6) with respect to $\beta_i$ gives

$$\begin{aligned}\frac{\partial^2 p_3}{\partial k_i \partial \beta_i} &\propto \big(\gamma^2\varepsilon^4\nu^2(\Omega + 2\Sigma) - \sqrt{\gamma^4\varepsilon^8\nu^4(\Omega + 2\Sigma)^2 - 4(\gamma^2\varepsilon^4\nu^2 - \eta^2 - \varepsilon^2)\Upsilon\Sigma^2\rho^2}\big) \\ &\times \Big(\frac{\gamma^2\varepsilon^4\nu^2(\Omega + 2\Sigma)^2}{\Upsilon\sigma^2\beta_i^2} + \frac{2\gamma^2\varepsilon^4\nu^2(\Omega + 2\Sigma)}{\gamma^2\varepsilon^4\nu^2 - \eta^2 - \varepsilon^2} - 4\sigma^2\beta_{\neg i}^2\Big) - 4\gamma^2\sigma^2\varepsilon^4\nu^2(\Omega + 2\Sigma)\beta_{\neg i}^2 \\ &+ \frac{2(\Omega + 2\Sigma)\Upsilon}{\gamma^2\varepsilon^4\nu^2 - \eta^2 - \varepsilon^2}\sqrt{\gamma^4\varepsilon^8\nu^4(\Omega + 2\Sigma)^2 - 4(\gamma^2\varepsilon^4\nu^2 - \eta^2 - \varepsilon^2)\Upsilon\Sigma^2\rho^2}.\end{aligned}$$

The first quantity is positive, and

$$\frac{2\gamma^2\varepsilon^4\nu^2(\Omega + 2\Sigma)}{\gamma^2\varepsilon^4\nu^2 - \eta^2 - \varepsilon^2} - 4\sigma^2\beta_{\neg i}^2 = \frac{2\gamma^2\varepsilon^4\nu^2[\Omega + 2(\Sigma - \sigma^2\beta_{\neg i}^2)] + 4(\eta^2 + \varepsilon^2)\sigma^2\beta_{\neg i}^2}{\gamma^2\varepsilon^4\nu^2 - \eta^2 - \varepsilon^2} > 0,$$

so the second quantity is also positive. Finally, the remaining portion of the expression is positive if and only if

$$\sqrt{\gamma^4\varepsilon^8\nu^4(\Omega + 2\Sigma)^2 - 4(\gamma^2\varepsilon^4\nu^2 - \eta^2 - \varepsilon^2)\Upsilon\Sigma^2\rho^2} > \frac{2\gamma^2\sigma^2\varepsilon^4\nu^2(\gamma^2\varepsilon^4\nu^2 - \eta^2 - \varepsilon^2)\beta_{\neg i}^2}{\Upsilon}.$$

Note that the left-hand side of this inequality is decreasing in both $\beta_i$ and $\beta_{\neg i}$ (because $\rho$ is increasing in both $\beta_i$ and $\beta_{\neg i}$) whereas the right-hand side is increasing in $\beta_{\neg i}$. Substituting (6), (9), and

$\beta_i = 1$ into the left-hand side and $\beta_{\neg i} = 1$ into both sides of the inequality, squaring both sides of the inequality, and subtracting the right-hand side from the left-hand side gives

$$\frac{\gamma^4\eta^2\varepsilon^{10}\nu^4[3\Upsilon + 2\gamma^2\eta^2\varepsilon^2\nu^2]}{(\eta^2+\varepsilon^2)\Upsilon}\Big(6\sigma^2 + \frac{\eta^2\varepsilon^2[3\Upsilon + 2\gamma^2\eta^2\varepsilon^2\nu^2]}{(\eta^2+\varepsilon^2)\Upsilon}\Big) \\ + 4(\eta^2+\varepsilon^2)^2\sigma^4 + \frac{\gamma^4\sigma^4\varepsilon^8\nu^4}{\Upsilon^2}[5\Upsilon^2 - 4(\gamma^2\varepsilon^4\nu^2 - \eta^2 - \varepsilon^2)^2] > 0.$$

Thus, $\partial p_3/\partial k_i$ is strictly increasing in $\beta_i$.

Next, differentiating (A.6) with respect to $\beta_{\neg i}$ gives

$$\frac{\partial^2 p_3}{\partial k_i \partial \beta_{\neg i}} = -\frac{2\sigma^4 \frac{\eta^2}{\eta^2+\varepsilon^2}(\gamma^2\varepsilon^4\nu^2 - \eta^2 - \varepsilon^2)^2\Upsilon[\Omega + 2(\Sigma - \sigma^2\beta_i^2)]\beta_1\beta_2\beta_3}{\sqrt{\gamma^4\varepsilon^8\nu^4(\Omega+2\Sigma)^2 - 4(\gamma^2\varepsilon^4\nu^2 - \eta^2 - \varepsilon^2)\Upsilon\Sigma^2\rho^2}} \\ \div \big[(\eta^2+\varepsilon^2)(\Omega+2\Sigma) + \sqrt{\gamma^4\varepsilon^8\nu^4(\Omega+2\Sigma)^2 - 4(\gamma^2\varepsilon^4\nu^2 - \eta^2 - \varepsilon^2)\Upsilon\Sigma^2\rho^2}\big]^2 < 0.$$

Therefore, $\partial p_3/\partial k_i$ is strictly decreasing in $\beta_{\neg i}$. $\square$

## Appendix B: Portfolio constrained informed traders

All three traders trade both stocks in the baseline model. That framework demonstrates that prices systematically incorporate idiosyncratic information, but the solutions are intricate and do not admit interpretable comparative statics. In this appendix, I analyze an economy where trader $i$ trades only stock $i$ to analytically confirm the numerical results presented in Section 3 and to show that idiosyncratic information is reflected in the price of another asset even when the trader with information does not trade the other asset Other models with market segmentation include, for instance, Merton (1987) and Huangfu and Liu (2025). All other assumptions are unchanged, but informed trader $i$ is endowed with $w_i^s$ shares of stock $i$ and no shares of stock $\neg i$.

When trading only one asset, informed trader $i$'s demand function is

$$s_i = \frac{\frac{\eta^2}{\eta^2+\varepsilon^2}k_i - \bar{\theta}k_{\neg i} - \psi_i + \gamma\bar{\delta}w_i^s}{\gamma(\Omega + 2\bar{\delta})}. \tag{B.1}$$

The uninformed trader's objective is unaffected by whether the other traders trade both stocks, and his demand is given by (20). The equilibrium prices are derived in the same way as described in Section 3, except that the market-clearing condition is $s_i + \hat{s}_i = 1$.

**Proposition B.1.** *The unique equilibrium price of stock $i$ is given by*

$$p_i = \bar{\lambda}_i + \bar{\xi}k_i + \bar{\phi}k_{\neg i}, \tag{B.2}$$

*where $\bar{\lambda}_i$, $\bar{\xi}$, and $\bar{\phi}$ are defined in Appendix A.*

Proposition B.1 shows that idiosyncratic private information is systematically incorporated into prices even when the trader who possess the information trades only the stock about which he is informed. The real advantage of this framework, however, is that it admits interpretable comparative statics. Define $\bar{\Gamma} \equiv |\bar{\phi}/\bar{\xi}|$ and $\bar{\Theta} \equiv |\frac{\partial s_i}{\partial k_{\neg i}}|/\frac{\partial s_i}{\partial k_i}|$.

**Corollary B.1.** *The degree to which $p_i$ and trader $i$'s allocation reflect payoff-irrelevant idiosyncratic information is increasing in the level of systematic risk, i.e., $\partial\bar{\Gamma}/\partial B > 0$ and $\partial\bar{\Theta}/\partial B > 0$.*

Corollary B.1 is consistent with the numerical results obtained in Section 3.

## Appendix C: Cross-sectional effects with two informed traders

The analysis in Section 4 explores the cross-sectional pricing effects of idiosyncratic information when there are three informed traders. Due to the inherent complexity, the equilibrium is derived numerically. In this appendix, I analyze a simplified version of the model that admits a unique equilibrium and permits closed-form analytic comparative statics. The results are consistent with those presented in Section 4.

I extend the model described in Appendix B by introducing a third stock, the payoff of which is given by (23). I assume that the third stock is traded only by the uninformed trader, who receives an endowment $\hat{w}_3^s = 1$. Assuming that only the uninformed trader trades the third stock simplifies the analysis and permits the derivation of analytic solutions from which economic insights can be gleaned. In this setting, informed traders are not directly affected by the existence of the third stock because they do not trade it. Hence, their objectives and demand functions are the same as those characterized in Appendix B. While the third stock affects the uninformed trader's demand for the other two stocks, which in turn affects price levels, it does not affect the sensitivity of prices to the informed traders' trades. Thus, the price of stock $i$ is conjectured (and verified) to be described by (3) with

$$\psi_i' \equiv \bar{\psi}_i - \frac{\gamma\sigma^2(\Omega+2\delta)(\Omega+\delta+\Sigma-\sigma^2\beta_{\neg i}^2)\beta_i\beta_3}{(\Omega+\delta+\Sigma)^2-\Sigma^2\rho^2}$$

in place of $\psi_i$. The equilibrium prices are derived in a similar fashion as those in Proposition 1.

**Proposition C.1.** *The unique equilibrium price of stock 3 is given by*

$$p_3 = -\gamma(\sigma^2+\eta^2) - \gamma\sigma^2(\beta_1+\beta_2)\beta_3 - \sigma^2\Big(\frac{\psi_1'-\gamma\delta w_1^s}{\Omega+2\delta}\beta_1 + \frac{\psi_2'-\gamma\delta w_2^s}{\Omega+2\delta}\beta_2\Big)\beta_3 + \frac{\sigma^2\big(\frac{\eta^2}{\eta^2+\varepsilon^2}\beta_1-\theta\beta_2\big)\beta_3}{\Omega+2\delta}k_1 + \frac{\sigma^2\big(\frac{\eta^2}{\eta^2+\varepsilon^2}\beta_2-\theta\beta_1\big)\beta_3}{\Omega+2\delta}k_2. \quad \text{(C.1)}$$

The expression for $p_3$ cleanly illustrates how idiosyncratic information about multiple assets is incorporated into the price of another asset. Consider private information about stock $i$. Trader $i$'s allocation increases by $\big(\frac{\eta^2}{\eta^2+\sigma^2}\big)/(\Omega+2\delta)$ in response to a marginal increase in $k_i$ (see (B.1)), which means that the uninformed trader's allocation in stock $i$ must decrease by an identical amount. This reduces the uninformed trader's systematic risk exposure by $\sigma^2\beta_i\big(\frac{\eta^2}{\eta^2+\sigma^2}\big)/(\Omega+2\delta)$. At the same time, his allocation in stock $\neg i$ increases by $\theta/(\Omega+2\delta)$ in response to a marginal increase in $k_i$, which raises his systematic risk exposure by $\sigma^2\beta_{\neg i}\theta/(\Omega+2\delta)$. The net change in his systematic risk exposure is $-\sigma^2\big(\frac{\eta^2}{\eta^2+\varepsilon^2}\beta_i-\theta\beta_{\neg i}\big)/(\Omega+2\delta)$. The uninformed trader adjusts his demand for the third stock to offset this change, resulting a price impact that is proportional to $\beta_3$ and the (negative of) the net change in his systematic risk exposure.

**Corollary C.1.** *The sensitivity of $p_3$ to idiosyncratic information about stock $i$'s payoff is increasing in stock $i$'s level of systematic risk and decreasing in stock $\neg i$'s level of systematic risk, i.e., $\partial^2 p_3/(\partial k_i \partial\beta_i) > 0$ and $\partial^2 p_3/(\partial k_i \partial\beta_{\neg i}) < 0$.*

Corollary C.1 is consistent with the numerical results obtained in Section 4.

**Table 1: Parameter values.**

| Variable | Symbol | Value |
|---|---|---|
| Risk aversion coefficient | $\gamma$ | 3 |
| Systematic volatility | $\sigma$ | 1 |
| Idiosyncratic volatility | $\eta$ | 1 |
| Information volatility | $\varepsilon$ | 3 |
| Nontradable endowment volatility | $\nu$ | 0.2 |
| Trader i's stock endowment | $w_i^s$ | 0 |
| Trader 3's stock endowment | $\hat{w}^s$ | 1 |

1 2

1. Traders receive their stock and bond endowments
2. Trader $i$ observes $z_i$ and $q_i$
3. Trade occurs

1. Stock $i$ pays $\pi_i$
2. Consumption occurs

**Figure 1: Timeline.**

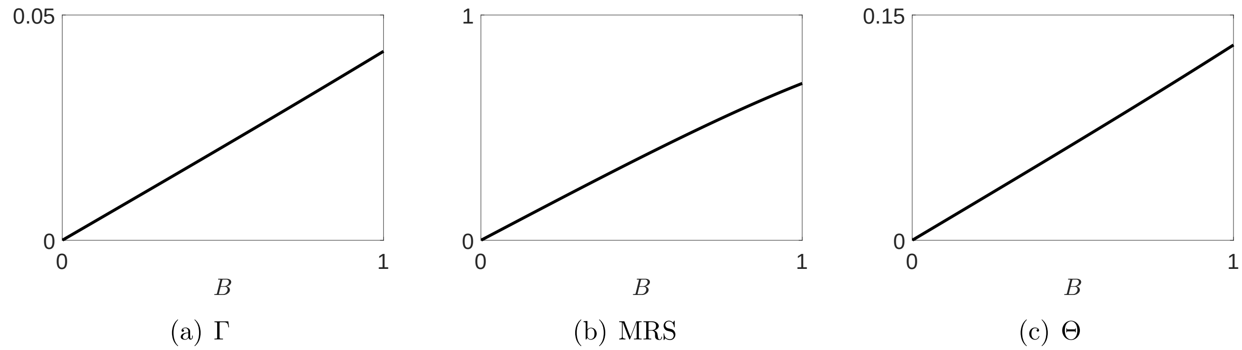
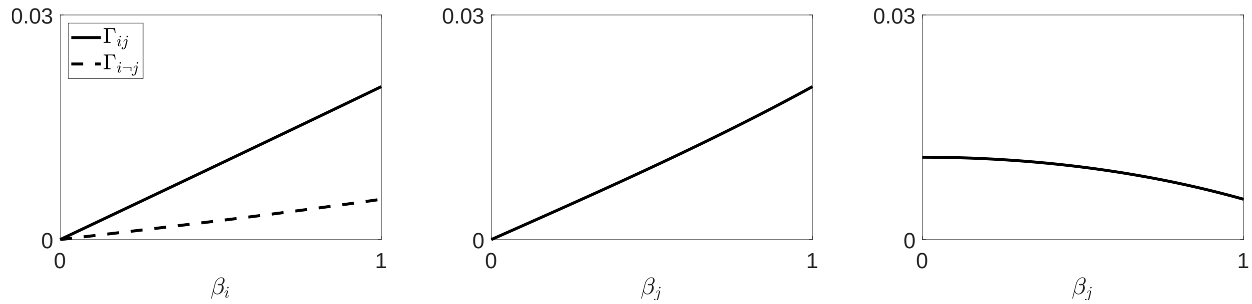



**Figure 2: Relative price impact.** The relative sensitivity of the price to payoff-irrelevant information compared to payoff-relevant information, $\Gamma$, is plotted in (a). The difference between an uninformed trader's and informed trader's marginal rate of substitution is plotted in (b). The relative sensitivity of an informed trader's equilibrium allocation to payoff-irrelevant information compared to payoff-relevant information, $\bar{\Theta}$, is plotted in (c).




(a) $\Gamma_{ij}$ & $\Gamma_{i\neg j}$ (b) $\Gamma_{ij}$ (c) $\Gamma_{i\neg j}$

**Figure 3: Cross-sectional price impact.** The relative sensitivity of $p_i$ to payoff-irrelevant information about stocks $j$ and $\neg j$ compared to payoff-relevant information about stock $i$, $\Gamma_{ij}$ and $\Gamma_{i\neg j}$, is plotted in (a) as a function of $\beta_i$. The relative sensitivity of $p_i$ to payoff-irrelevant information about stock $j$ compared to payoff-relevant information about stock $i$, $\Gamma_{ij}$, is plotted in (b) as a function of $\beta_j$. The relative sensitivity of the $p_i$ to payoff-irrelevant information about stock $\neg j$ compared to payoff-relevant information about stock $i$, $\Gamma_{i\neg j}$, is plotted in (c) as a function of $\beta_j$.